

## **COST-VOLUME-PROFIT RELATIONSHIPS**

Cost-volume-profit (CVP) analysis is concerned with the effects on net operating income of:

- Selling prices.
- Sales volume.
- Unit variable costs.
- Total fixed costs.
- The mix of products sold.

### **AGENDA**

- A. Review of contribution income statement.
- B. Effects of changes in sales volume on net operating income.
- C. CVP relationships in equation form
- D. CVP and profit graphs.
- E. Contribution margin (CM) ratio.
- F. Target profit analysis.
- G. Break-even analysis.
- H. Margin of safety.
- I. Operating leverage.
- J. Multiproduct break-even analysis.

## THE CONTRIBUTION APPROACH

A contribution format income statement is very useful in CVP analysis because it highlights cost behavior.

EXAMPLE: Last month's contribution income statement for Nord Corporation, a manufacturer of exercise bicycles, follows:

	<i>Total</i>	<i>Per Unit</i>
Sales ( <i>500 bikes</i> ) .....	\$250,000	\$500
Variable expenses .....	<u>150,000</u>	<u>300</u>
Contribution margin .....	100,000	<u>\$200</u>
Fixed expenses .....	<u>80,000</u>	
Net operating income.....	<u>\$ 20,000</u>	

### CONTRIBUTION MARGIN:

- The amount that sales (net of variable expenses) contributes toward covering fixed expenses and then toward profits.
- The unit contribution margin remains constant so long as the selling price and the unit variable cost do not change.

## VOLUME CHANGES AND NET OPERATING INCOME

Contribution income statements are given on this and the following page for monthly sales of 1, 2, 400, and 401 bikes.

	<i>Total</i>	<i>Per Unit</i>
Sales ( <i>1 bike</i> ) .....	\$ 500	\$500
Variable expenses .....	<u>300</u>	<u>300</u>
Contribution margin .....	200	<u>\$200</u>
Fixed expenses .....	<u>80,000</u>	
Net operating income (loss) ..	<u>\$(79,800)</u>	

	<i>Total</i>	<i>Per Unit</i>
Sales ( <i>2 bikes</i> ) .....	\$ 1,000	\$500
Variable expenses .....	<u>600</u>	<u>300</u>
Contribution margin .....	400	<u>\$200</u>
Fixed expenses .....	<u>80,000</u>	
Net operating income (loss) ..	<u>\$(79,600)</u>	

Note the following points:

1. The contribution margin must first cover the fixed expenses. If it doesn't, there is a loss.
2. As additional units are sold, fixed expenses are whittled down until they have all been covered.

**VOLUME CHANGES AND NET OPERATING INCOME (continued)**

	<i>Total</i>	<i>Per Unit</i>
Sales (400 bikes) .....	\$200,000	\$500
Variable expenses .....	<u>120,000</u>	<u>300</u>
Contribution margin .....	80,000	<u>\$200</u>
Fixed expenses .....	<u>80,000</u>	
Net operating income (loss)....	<u>\$ 0</u>	

	<i>Total</i>	<i>Per Unit</i>
Sales (401 bikes) .....	\$200,500	\$500
Variable expenses .....	<u>120,300</u>	<u>300</u>
Contribution margin .....	80,200	<u>\$200</u>
Fixed expenses .....	<u>80,000</u>	
Net operating income (loss)....	<u>\$ 200</u>	

Note the following points:

1. If the company sells exactly 400 bikes a month, it will just break even (no profit or loss).
2. The break-even point is:
  - The point where total sales revenue equals total expenses (variable and fixed).
  - The point where total contribution margin equals total fixed expenses.
3. Each additional unit sold increases net operating income by the amount of the unit contribution margin.

**CVP RELATIONSHIPS IN EQUATION FORM**

**Terry's Way shown in class:**

$$\text{Sales} = \text{Variable} + \text{Fixed} + \text{Net income}$$

$$\$500X = \$300X + \$80,000 + \$0$$

$$\$200X = \$80,000$$

$$\frac{\$200X}{\$200} = \frac{\$80,000}{\$200}$$

$$X = 400$$

**X = 400 bicycles @break even**

$$\text{Sales} = \text{Variable} + \text{Fixed} + \text{Net income}$$

$$\$500X = \$300X + \$80,000 + \$70,000$$

$$\$200X = \$150,000$$

$$\frac{\$200X}{\$200} = \frac{\$150,000}{\$200}$$

$$X = 750$$

**X = 750 bicycles @break even + \$70,000 profit**

**Garrison Textbook Way:**

The contribution format income statement can be expressed in equation form as follows:

$$\text{Profit} = (\text{Sales} - \text{Variable expense}) - \text{Fixed expense}$$

When a company has a single product, we can further refine the equation as follows:

$$\text{Profit} = (P \times Q - V \times Q) - \text{Fixed expense}$$

EXAMPLE: This equation can be used to compute Nord Company's net operating income if it sells 401 bikes:

$$\begin{aligned} \text{Profit} &= (\$500 \times 401 - \$300 \times 401) - \$80,000 \\ &= (\$500 - \$300) \times 401 - \$80,000 \\ &= (\$200) \times 401 - \$80,000 \\ &= \$80,200 - \$80,000 = \$200 \end{aligned}$$

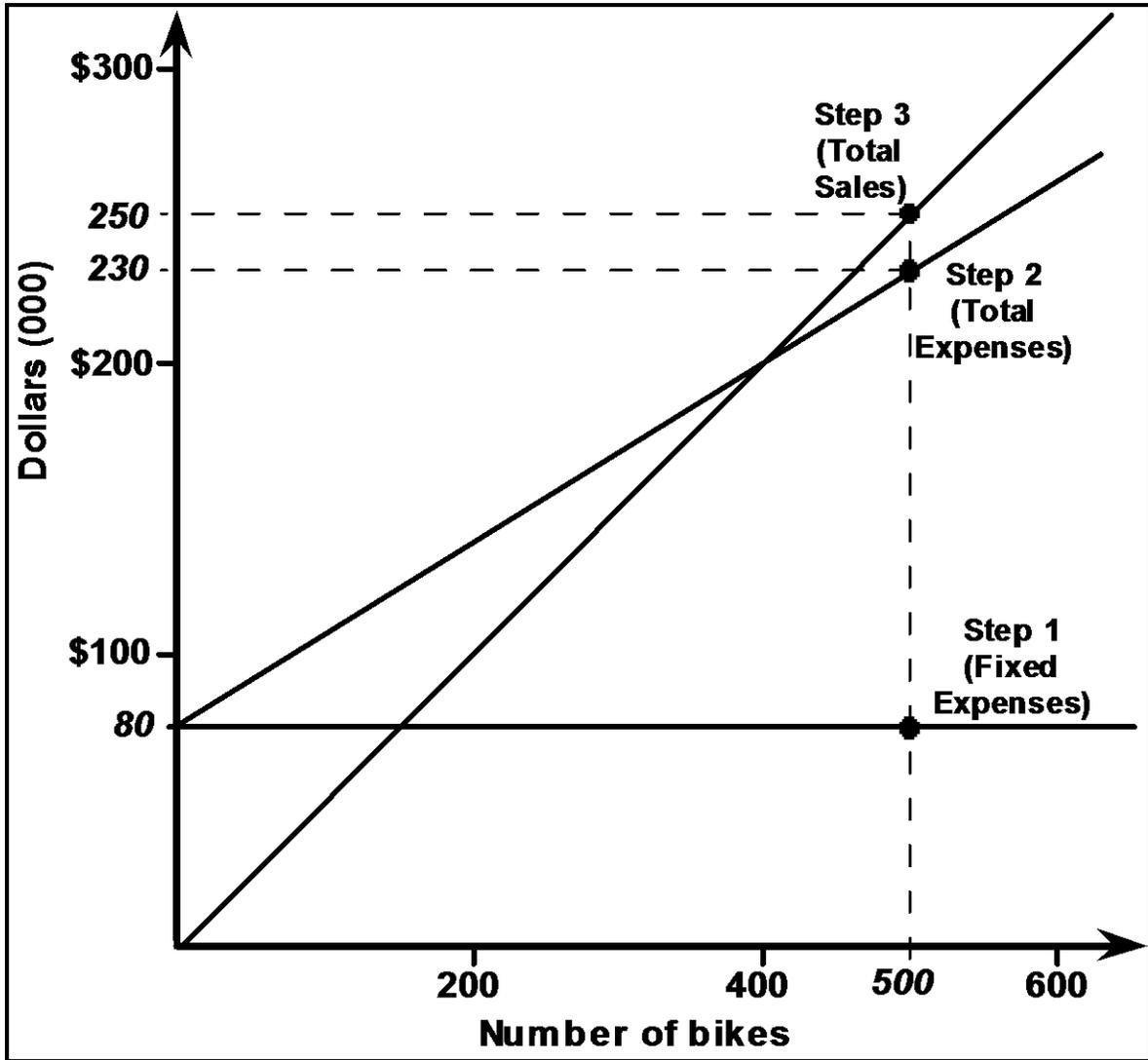
It is often useful to express the simple profit equation in terms of the unit contribution margin as follows:

$$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed expense}$$

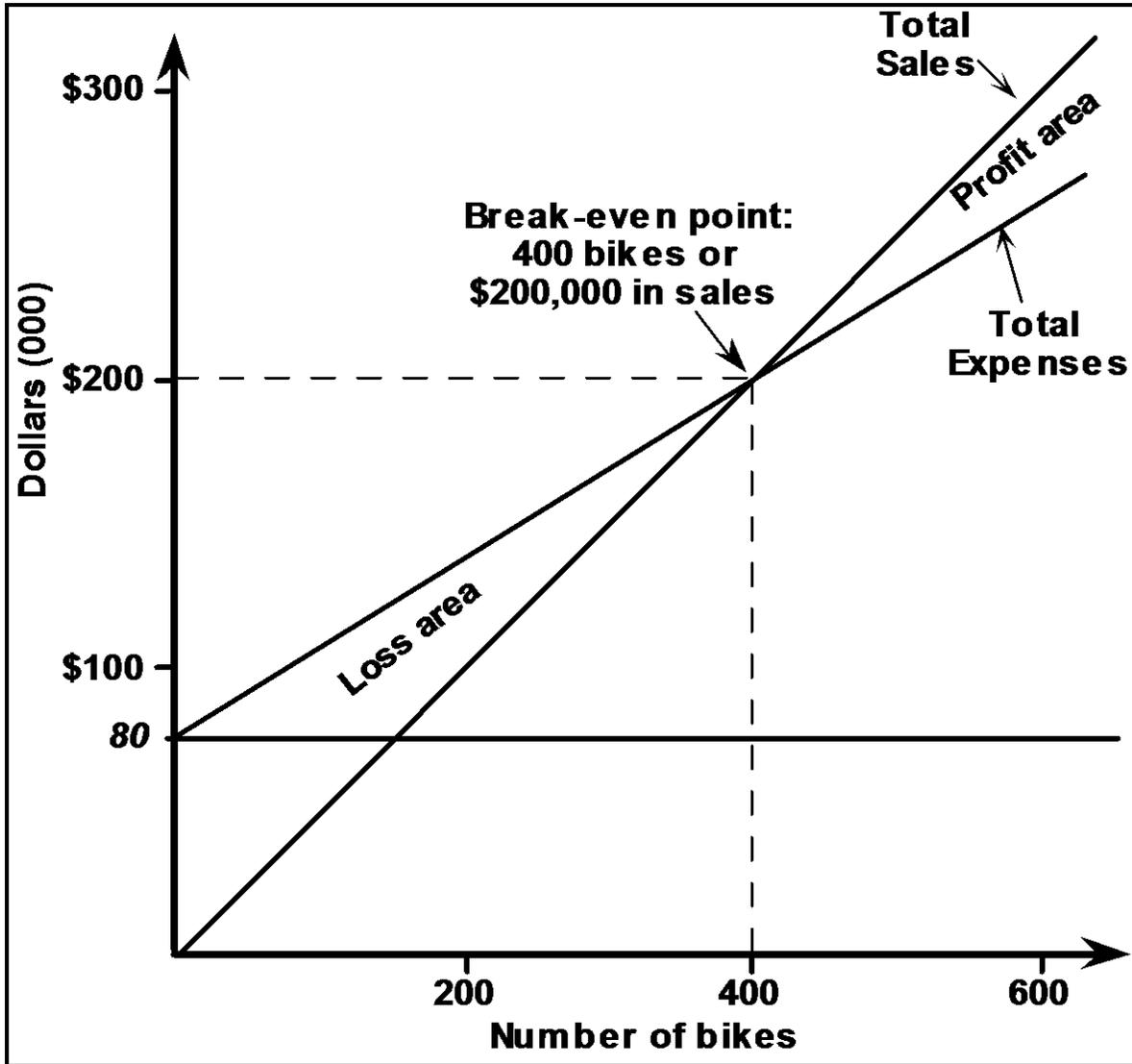
This equation can also be used to compute Nord Company's net operating income if it sells 401 bikes:

$$\begin{aligned} \text{Profit} &= \$200 \times 401 - \$80,000 \\ \text{Profit} &= \$80,200 - \$80,000 = \$200 \end{aligned}$$

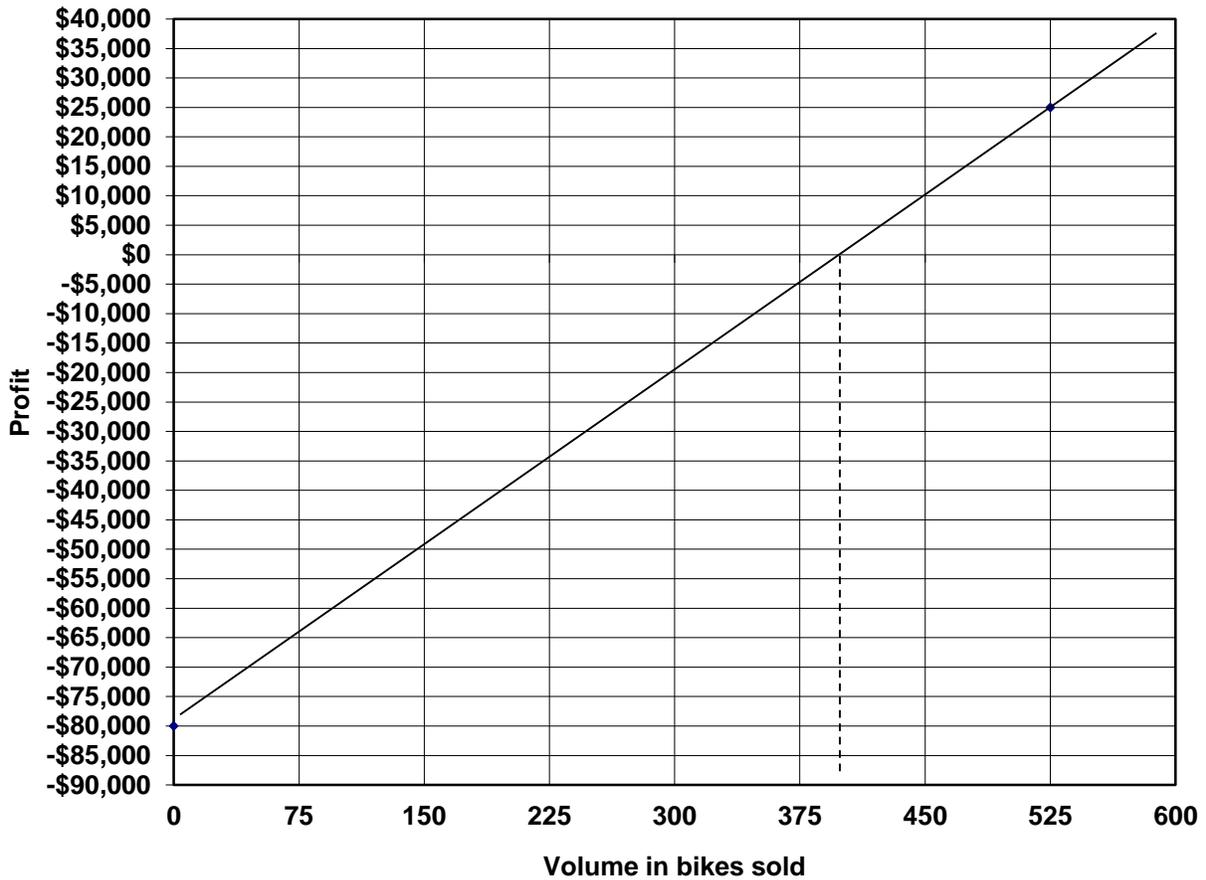
### PREPARING A CVP GRAPH



### THE COMPLETED CVP GRAPH



### THE PROFIT GRAPH



Notice, the straight line goes through two data points at (0, -\$80,000) and (525, \$25,000). The break-even point is denoted by the dotted line at 400 bikes.

## CONTRIBUTION MARGIN RATIO

The contribution margin (CM) ratio is the ratio of contribution margin to total sales:

$$\text{CM ratio} = \frac{\text{Contribution margin}}{\text{Total sales}}$$

If the company has only one product, the CM ratio can also be computed using per unit data:

$$\text{CM ratio} = \frac{\text{Unit contribution margin}}{\text{Unit selling price}}$$

EXAMPLE: For Nord Corporation, the CM ratio is 40%, computed as follows:

$$\text{CM ratio} = \frac{\text{Contribution margin}}{\text{Total sales}} = \frac{\$100,000}{\$250,000} = 40\%$$

or

$$\text{CM ratio} = \frac{\text{Unit contribution margin}}{\text{Unit selling price}} = \frac{\$200 \text{ per unit}}{\$500 \text{ per unit}} = 40\%$$

The relation between profit and the CM ratio can also be expressed using the following equation:

$$\text{Profit} = \text{CM ratio} \times \text{Sales} - \text{Fixed expense}$$

EXAMPLE: Nord Company's profit if it sells 401 bikes can be computed as follows:

$$\text{Profit} = 0.40 \times \$200,500 - \$80,000$$

$$\text{Profit} = \$80,200 - \$80,000 = \$200$$

**CONTRIBUTION MARGIN RATIO (continued)**

The CM ratio shows how the contribution margin will be affected by a given change in total sales.

EXAMPLE: Assume that Nord Corporation’s sales increase by \$150,000 next month. What will be the effect on (1) the contribution margin and (2) net operating income?

(1) Effect on contribution margin:

$$\begin{aligned} \text{Increase in contribution margin} &= \text{Increase in sales} \times \text{CM ratio} \\ &= \$150,000 \times 40\% = \$60,000 \end{aligned}$$

(2) Effect on net operating income:

If fixed expenses do not change, the net operating income for the month will also increase by \$60,000.

	<i>Present</i>	<i>Expected</i>	<i>Change</i>
Sales (in units) .....	<u>500</u>	<u>800</u>	<u>300</u>
Sales (in dollars).....	\$250,000	\$400,000	\$150,000
Variable expenses.....	<u>150,000</u>	<u>240,000</u>	<u>90,000</u>
Contribution margin.....	100,000	160,000	60,000
Fixed expenses.....	<u>80,000</u>	<u>80,000</u>	<u>0</u>
Net operating income.....	<u>\$ 20,000</u>	<u>\$ 80,000</u>	<u>\$ 60,000</u>

## TARGET PROFIT ANALYSIS

### *Summary of Nord Corporation Data:*

	Per Bike	Percent	Per Month
Selling price .....	\$500	100%	
Variable expenses .....	<u>300</u>	<u>60%</u>	
Contribution margin .....	<u>\$200</u>	<u>40%</u>	
Fixed expenses .....			\$80,000

EXAMPLE: Assume that Nord Corporation's target profit is \$70,000 per month. How many exercise bikes must it sell to reach this goal?

### **EQUATION METHOD (Unit sales)**

**Q = Number of bikes to attain the target profit**

$$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed Expenses}$$

$$\$70,000 = \$200 \times Q - \$80,000$$

$$Q = 750 \text{ Bikes}$$

### **EQUATION METHOD (Sales dollars)**

What if Nord wanted to know how much sales revenue needed to be generated to achieve a target profit of \$70,000? We can compute the answer two ways. First, we can multiply the answer from above by the selling price per bike:

$$750 \text{ bikes} \times \$500 \text{ per bike} = \$375,000$$

Or, we can use the following equation:

$$\text{Profit} = \text{CM ratio} \times \text{Sales} - \text{Fixed Expenses}$$

$$\$70,000 = 0.40 \times \text{Sales} - \$80,000$$

$$\text{Sales} = \$375,000$$

**TARGET PROFIT ANALYSIS (continued)**

*Summary of Nord Corporation Data:*

	<i>Per Bike</i>	<i>Percent</i>	<i>Per Month</i>
Selling price .....	\$500	100%	
Variable expenses .....	<u>300</u>	<u>60%</u>	
Contribution margin .....	<u>\$200</u>	<u>40%</u>	
Fixed expenses .....			\$80,000

EXAMPLE: Assume that Nord Corporation’s target profit is \$70,000 per month. How many exercise bikes must it sell to reach this goal?

**FORMULA METHOD (Unit sales)**

$$\text{Unit sales to attain a target profit} = \frac{\text{Target profit} + \text{Fixed expense}}{\text{Unit CM}}$$

$$\text{Unit sales to attain a target profit} = \frac{\$70,000 + \$80,000}{\$200 \text{ per bike}} = 750 \text{ bikes}$$

**FORMULA METHOD (Dollar sales)**

What if Nord wanted to know how much sales revenue needed to be generated to achieve a target profit of \$70,000? We can use the following formula to derive the answer:

$$\text{Dollar sales to breakeven} = \frac{\text{Target profit} + \text{Fixed expense}}{\text{CM ratio}}$$

$$\text{Dollar sales to breakeven} = \frac{\$70,000 + \$80,000}{0.40} = \$200,000$$

**BREAK-EVEN ANALYSIS***Summary of Nord Corporation Data:*

	<i>Per Bike</i>	<i>Percent</i>	<i>Per Month</i>
Selling price .....	\$500	100%	
Variable expenses .....	<u>300</u>	<u>60%</u>	
Contribution margin .....	<u>\$200</u>	<u>40%</u>	
Fixed expenses .....			\$80,000

EXAMPLE: Using the equation method, we can compute Nord Company's break-even point in unit sales as follows:

**EQUATION METHOD (Unit sales)**

$$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed expense}$$

$$\$0 = \$200 \times Q - \$80,000$$

$$\$200 \times Q = \$0 + \$80,000$$

$$Q = (\$0 + \$80,000) \div \$200$$

$$Q = 400 \text{ bikes}$$

EXAMPLE: Using the equation method, we can compute Nord Company's break-even point in dollar sales as follows:

**EQUATION METHOD (Dollar sales)**

$$\text{Profit} = \text{CM ratio} \times \text{Sales} - \text{Fixed Expenses}$$

$$\$0 = 0.40 \times \text{Sales} - \$80,000$$

$$0.40 \times \text{Sales} = \$0 + \$80,000$$

$$\text{Sales} = (\$0 + \$80,000) \div 0.40$$

$$\text{Sales} = \$200,000$$

**BREAK-EVEN ANALYSIS (continued)**

*Summary of Nord Corporation Data:*

	<i>Per Bike</i>	<i>Percent</i>	<i>Per Month</i>
Selling price .....	\$500	100%	
Variable expenses .....	<u>300</u>	<u>60%</u>	
Contribution margin .....	<u>\$200</u>	<u>40%</u>	
Fixed expenses .....			\$80,000

EXAMPLE: Using the formula method, we can compute Nord Company's break-even point in unit sales as follows:

**FORMULA METHOD (Units sales)**

$$\text{Unit sales to breakeven} = \frac{\text{Fixed expense}}{\text{Unit CM}}$$

$$\text{Unit sales to breakeven} = \frac{\$80,000}{\$200 \text{ per bike}} = 400 \text{ bikes}$$

**FORMULA METHOD (Dollar sales)**

EXAMPLE: Using the formula method, we can compute Nord Company's break-even point in dollar sales as follows:

$$\text{Dollar sales to breakeven} = \frac{\text{Fixed expense}}{\text{CM ratio}}$$

$$\text{Dollar sales to breakeven} = \frac{\$80,000}{0.40} = \$200,000$$

### MARGIN OF SAFETY

The margin of safety is the excess of budgeted (or actual) sales over the break-even sales. The margin of safety can be expressed either in dollar or percentage form. The formulas are:

$$\text{Margin of safety in dollars} = \text{Total sales} - \text{Breakeven sales}$$

$$\text{Margin of safety percentage} = \frac{\text{Margin of safety in dollars}}{\text{Total sales}}$$

	<i>Company X</i>		<i>Company Y</i>	
Sales .....	\$500,000	100%	\$500,000	100%
Variable expenses.....	<u>350,000</u>	<u>70%</u>	<u>100,000</u>	<u>20%</u>
Contribution margin.....	150,000	<u>30%</u>	400,000	<u>80%</u>
Fixed expenses.....	<u>90,000</u>		<u>340,000</u>	
Net operating income .....	<u>\$ 60,000</u>		<u>\$ 60,000</u>	
Break-even point:				
\$90,000 ÷ 0.30.....	\$300,000			
\$340,000 ÷ 0.80.....			\$425,000	
Margin of safety in dollars:				
\$500,000 – \$300,000 .....	\$200,000			
\$500,000 – \$425,000 .....			\$75,000	
Margin of safety percentage:				
\$200,000 ÷ \$500,000.....	40%			
\$75,000 ÷ \$500,000 .....			15%	

### OPERATING LEVERAGE

Operating leverage measures how a given percentage change in sales affects net operating income.

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Net operating income}}$$

	<i>Company X</i>		<i>Company Y</i>	
Sales .....	\$500,000	100%	\$500,000	100%
Variable expenses.....	<u>350,000</u>	<u>70%</u>	<u>100,000</u>	<u>20%</u>
Contribution margin.....	150,000	<u>30%</u>	400,000	<u>80%</u>
Fixed expenses.....	<u>90,000</u>		<u>340,000</u>	
Net operating income .....	<u>\$ 60,000</u>		<u>\$ 60,000</u>	
Degree of operating leverage .	2.5		6.7	

If the degree of operating leverage is 2.5, then a 10% increase in sales should result in a 25% (= 2.5 × 10%) increase in net operating income.

EXAMPLE: Assume that both company X and company Y experience a 10% increase in sales:

	<i>Company X</i>		<i>Company Y</i>	
Sales .....	\$550,000	100%	\$550,000	100%
Variable expenses.....	<u>385,000</u>	<u>70%</u>	<u>110,000</u>	<u>20%</u>
Contribution margin.....	165,000	<u>30%</u>	440,000	<u>80%</u>
Fixed expenses.....	<u>90,000</u>		<u>340,000</u>	
Net operating income .....	<u>\$ 75,000</u>		<u>\$100,000</u>	
Increase in net operating income..	25%		67%	

### OPERATING LEVERAGE (continued)

The degree of operating leverage is not constant—it changes with the level of sales.

EXAMPLE: At the higher level of sales, the degree of operating leverage for Company X decreases from 2.5 to 2.2 and for Company Y from 6.7 to 4.4.

	<i>Company X</i> <i>(000s)</i>		<i>Company Y</i> <i>(000s)</i>	
Sales .....	\$500	\$550	\$500	\$550
Variable expenses .....	<u>350</u>	<u>385</u>	<u>100</u>	<u>110</u>
Contribution margin .....	150	165	400	440
Fixed expenses .....	<u>90</u>	<u>90</u>	<u>340</u>	<u>340</u>
Net operating income .....	<u>\$ 60</u>	<u>\$ 75</u>	<u>\$ 60</u>	<u>\$100</u>
Degree of operating leverage.....	2.5	2.2	6.7	4.4

Ordinarily, the degree of operating leverage declines as sales increase.

### MULTIPRODUCT BREAK-EVEN ANALYSIS

When a company has multiple products, the overall contribution margin (CM) ratio is used in break-even analysis.

$$\text{Overall CM ratio} = \frac{\text{Total contribution margin}}{\text{Total sales dollars}}$$

	<i>Product A</i>		<i>Product B</i>		<i>Total</i>	
Sales .....	\$100,000	100%	\$300,000	100%	\$400,000	100.0%
Variable expenses.....	70,000	70%	120,000	40%	190,000	47.5%
Contribution margin.....	<u>\$ 30,000</u>	<u>30%</u>	<u>\$180,000</u>	<u>60%</u>	210,000	<u>52.5%</u>
Fixed expenses.....					<u>141,750</u>	
Net operating income ...					<u>\$ 68,250</u>	

$$\text{Overall CM ratio} = \frac{\text{Total contribution margin}}{\text{Total sales dollars}} = \frac{\$210,000}{\$400,000} = 52.5\%$$

$$\text{Breakeven sales} = \frac{\text{Fixed expenses}}{\text{Overall CM ratio}} = \frac{\$141,750}{0.525} = \$270,000$$

**MULTIPRODUCT BREAK-EVEN ANALYSIS (continued)**

The relative proportions in which the products are sold is called the sales mix. If the sales mix changes, the overall contribution margin ratio will change.

Example: Assume that total sales remain unchanged at \$400,000. However, the sales mix shifts so that more of Product A is sold than of Product B.

	<i>Product A</i>		<i>Product B</i>		<i>Total</i>	
Sales .....	\$300,000	100%	\$100,000	100%	\$400,000	100.0%
Variable expenses.....	210,000	<u>70%</u>	<u>40,000</u>	<u>40%</u>	<u>250,000</u>	<u>62.5%</u>
Contribution margin.....	<u>\$ 90,000</u>	<u>30%</u>	<u>\$ 60,000</u>	<u>60%</u>	150,000	<u>37.5%</u>
Fixed expenses.....					<u>141,750</u>	
Net operating income ...					<u>\$ 8,250</u>	

$$\text{Overall CM ratio} = \frac{\text{Total contribution margin}}{\text{Total sales dollars}} = \frac{\$150,000}{\$400,000} = 37.5\%$$

$$\text{Breakeven sales} = \frac{\text{Fixed expenses}}{\text{Overall CM ratio}} = \frac{\$141,750}{0.375} = \$378,000$$

## **MAJOR ASSUMPTIONS OF CVP ANALYSIS**

1. Selling price is constant. The price does not change as volume changes.
2. Costs are linear and can be accurately split into fixed and variable elements. The total fixed cost is constant and the variable cost per unit is constant.
3. The sales mix is constant in multi-product companies.
4. In manufacturing companies, inventories do not change. The number of units produced equals the number of units sold.